

# 7.1

## Ratio and Proportion

**Goal** Use ratios and proportions.

### VOCABULARY

**Ratio** A ratio is a comparison of a number  $a$  and a nonzero number  $b$  using division.

**Proportion** An equation that states that two ratios are equal is called a proportion.

**Means** In the proportion  $\frac{a}{b} = \frac{c}{d}$ , the numbers  $b$  and  $c$  are the means of the proportion.

**Extremes** In the proportion  $\frac{a}{b} = \frac{c}{d}$ , the numbers  $a$  and  $d$  are the extremes of the proportion.

### Example 1 Simplify Ratios

Simplify the ratio.

a. 60 cm : 200 cm

b.  $\frac{3 \text{ ft}}{18 \text{ in.}}$

#### Solution

a. 60 cm : 200 cm can be written as the fraction  $\frac{60 \text{ cm}}{200 \text{ cm}}$ .

Divide the numerator and the denominator by their greatest common factor, 20.

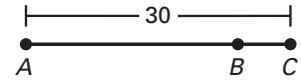
$$\frac{60 \text{ cm}}{200 \text{ cm}} = \frac{60 \div 20}{200 \div 20} = \frac{3}{10}$$

b. Before you can simplify this ratio, the quantities in the numerator and denominator must be written in the same units. To use inches, substitute 12 in. for 1 ft.

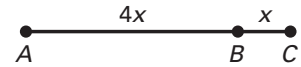
$$\frac{3 \text{ ft}}{18 \text{ in.}} = \frac{3 \cdot 12 \text{ in.}}{18 \text{ in.}} = \frac{36 \text{ in.}}{18 \text{ in.}} = \frac{36 \div 18}{18 \div 18} = \frac{2}{1}$$

**Example 2**    *Use Ratios*

In the diagram,  $AB : BC$  is  $4 : 1$  and  $AC = 30$ . Find  $AB$  and  $BC$ .

**Solution**

Let  $x = BC$ . Because the ratio of  $AB$  to  $BC$  is 4 to 1, you know  $AB = 4x$ .



$$AB + BC = AC \quad \text{Segment Addition Postulate}$$

$$4x + x = 30 \quad \text{Substitute.}$$

$$5x = 30 \quad \text{Add like terms.}$$

$$x = 6 \quad \text{Divide each side by 5.}$$

To find  $AB$  and  $BC$ , substitute your value for  $x$ .

$$AB = 4x = 4(6) = 24 \quad BC = x = 6$$

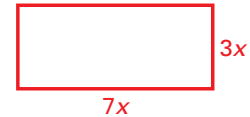
Answer  $AB = 24$  and  $BC = 6$ .

**Example 3**    *Use Ratios*

The perimeter of a rectangle is 80 feet. The ratio of the length to the width is  $7 : 3$ . Find the length and the width.

**Solution**

The ratio of length to width is  $7$  to  $3$ , you can let the length  $l = 7x$  and the width  $w = 3x$ . Draw the rectangle at the right and label side lengths.



$$2l + 2w = P \quad \text{Formula for perimeter of a rectangle}$$

$$2(7x) + 2(3x) = 80 \quad \text{Substitute.}$$

$$14x + 6x = 80 \quad \text{Multiply.}$$

$$20x = 80 \quad \text{Add like terms.}$$

$$x = 4 \quad \text{Divide each side by 20.}$$

To find length and width, substitute your value for  $x$ .

$$l = 7x = 7(4) = 28 \quad w = 3x = 3(4) = 12$$

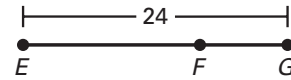
Answer The length is  $28$  feet and the width is  $12$  feet.

**Follow-Up** Check your answer for Example 3.

$$2l + 2w = 2(\underline{28}) + 2(\underline{12}) = \underline{80} \quad P = \underline{80}$$

✓ **Checkpoint** Complete the following exercises.

1. In the diagram,  $EF : FG$  is  $2 : 1$  and  $EG = 24$ . Find  $EF$  and  $FG$ .



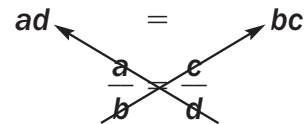
$$EF = 16; FG = 8$$

2. The perimeter of a rectangle is 84 feet. The ratio of the length to the width is  $4 : 3$ . Find the length and the width of the rectangle.

$$\text{length} = 24 \text{ ft}; \text{width} = 18 \text{ ft}$$

### CROSS PRODUCT PROPERTY

**Words** In a proportion, the product of the extremes is equal to the product of the means.



**Symbols** If  $\frac{a}{b} = \frac{c}{d}$ , then  $ad$  =  $bc$ .

**Example 4****Solve a Proportion**

Solve the proportion  $\frac{5}{3} = \frac{y + 2}{6}$ .

**Solution**

$$\frac{5}{3} = \frac{y + 2}{6}$$

Write the original proportion.

$$(5)(\underline{6}) = (3)(\underline{y + 2})$$

Cross product property

$$\underline{30} = \underline{3y + 6}$$

Multiply and use distributive property.

$$\underline{24} = \underline{3y}$$

Subtract 6 from each side.

$$\underline{8} = \underline{y}$$

Divide each side by 3.

✓ **Checkpoint** Solve the proportion.

3.  $\frac{3}{x} = \frac{6}{8}$

4

4.  $\frac{5}{3} = \frac{15}{y}$

9

5.  $\frac{m + 2}{5} = \frac{14}{10}$

5